Student ID:

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③	③	③	③	③	③
④	④	④	④	④	④
⑤	⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨	⑨

*Instructions: fill **completely** the bubbles with the digits of the SID (one for each column); in the lower part of the sheet, fill **completely** the bubbles with the correct answers to the corresponding question. Use a black or dark blue pen or pencil, trying to fill completely the inside of the bubble. Write only in the designated areas.*

Last Name:.......................... **First name**:.......................... **Signature**:..........................

Mark the answers of the multiple-choice questions

(1) Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ
(2) Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ
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(12) Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ
(13) Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ
(14) Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

(1) Let X and Y be statements. If we know that X implies Y, then we can also conclude that

- ☐ *(a)* X is true, and Y is also true.
- ☐ *(b)* Y cannot be false.
- ☐ *(c)* If Y is true, then X is true.
- ☐ *(d)* If Y is false, then X is false.
- ☐ *(e)* If X is false, then Y is false.
- ☐ *(f)* X cannot be false.
- ☐ *(g)* At least one of X and Y is true.

(2) Let X and Y be statements. If we want to DISPROVE the claim that "Both X and Y are true", we need to show that

- ☐ *(a)* At least one of X and Y are false.
- ☐ *(b)* X and Y are both false.
- ☐ *(c)* X is false.
- ☐ *(d)* Y is false.
- ☐ *(e)* X does not imply Y, and Y does not imply X.
- ☐ *(f)* Exactly one of X and Y are false.
- ☐ *(g)* X is true if and only if Y is false.

(3) Let X and Y be statements. If we want to DISPROVE the claim that "At least one of X and Y are true", we need to show that

- ☐ *(a)* At least one of X and Y are false.
- ☐ *(b)* X and Y are both false.
- ☐ *(c)* X is false.
- ☐ *(d)* Y is false.
- ☐ *(e)* X does not imply Y, and Y does not imply X.
- ☐ *(f)* Exactly one of X and Y are false.
- ☐ *(g)* X is true if and only if Y is false.

(4) Let X and Y be statements. If we want to DISPROVE the claim that "$X \implies Y$", we need to show that

- ☐ *(a)* Y is true, but X is false.
- ☐ *(b)* X is true, but Y is false.
- ☐ *(c)* X is false.
- ☐ *(d)* Y is false.
- ☐ *(e)* X and Y are both false.
- ☐ *(f)* Exactly one of X and Y are false.
- ☐ *(g)* At least one of X and Y is false.

(5) Let $P(x)$ be a property about some object x of type X. If we want to DISPROVE the claim that "$P(x)$ is true for all x of type X", then we have to

- ☐ *(a)* Show that there exists an x of type X for which $P(x)$ is false.
- ☐ *(b)* Show that there exists an x which is not of type X, but for which $P(x)$ is still true.
- ☐ *(c)* Show that for every x in X, $P(x)$ is false.
- ☐ *(d)* Show that $P(x)$ being true does not necessarily imply that x is of type X.
- ☐ *(e)* Assume there exists an x of type X for which $P(x)$ is true, and derive a contradiction.
- ☐ *(f)* Show that there are no objects x of type X.
- ☐ *(g)* Show that for every x in X, there is a y not equal to x for which $P(y)$ is true.

(6) Let $P(x)$ be a property about some object x of type X. If we want to DISPROVE the claim that "$P(x)$ is true for some x of type X", then we have to

- ☐ *(a)* Show that there exists an x of type X for which $P(x)$ is false.
- ☐ *(b)* Show that there exists an x which is not of type X, but for which $P(x)$ is still true.
- ☐ *(c)* Show that for every x in X, $P(x)$ is false.
- ☐ *(d)* Show that $P(x)$ being true does not necessarily imply that x is of type X.
- ☐ *(e)* Assume that $P(x)$ is true for every x in X, and derive a contradiction.
- ☐ *(f)* Show that there are no objects x of type X.
- ☐ *(g)* Show that for every x in X, there is a y not equal to x for which $P(y)$ is true.

(7) Let $P(n, m)$ be a property about two integers n and m. If we want to prove that "For every integer n, there exists an integer m such that $P(n, m)$ is true", then we should do the following:

- ☐ *(a)* Let n be an arbitrary integer. Then find an integer m (possibly depending on n) such that $P(n, m)$ is true.
- ☐ *(b)* Let n and m be arbitrary integers. Then show that $P(n, m)$ is true.
- ☐ *(c)* Find an integer n and an integer m such that $P(n, m)$ is true.
- ☐ *(d)* Let m be an arbitrary integer. Then find an integer n (possibly depending on m) such that $P(n, m)$ is true.
- ☐ *(e)* Find an integer n such that $P(n, m)$ is true for every integer m.
- ☐ *(f)* Find an integer m such that $P(n, m)$ is true for every integer n.
- ☐ *(g)* Show that whenever $P(n, m)$ is true, then n and m are integers.

(8) Let $P(n, m)$ be a property about two integers n and m. If we want to DISPROVE the claim that "For every integer n, there exists an integer m such that $P(n, m)$ is true", then we need to prove that

- ☐ *(a)* There exists an integer n such that $P(n, m)$ is false for all integers m.
- ☐ *(b)* There exists integers n,m such that $P(n, m)$ is false.
- ☐ *(c)* For every integer n, and every integer m, the property $P(n, m)$ is false.
- ☐ *(d)* For every integer n, there exists an integer m such that $P(n, m)$ is false.
- ☐ *(e)* For every integer m, there exists an integer n such that $P(n, m)$ is false.
- ☐ *(f)* There exists an integer m such that $P(n, m)$ is false for all integers n.
- ☐ *(g)* If $P(n, m)$ is true, then n and m are not integers.

(9) Let $P(n, m)$ be a property about two integers n and m. If we want to DISPROVE the claim that "There exists an integer n such that $P(n, m)$ is true for all integers m", then we need to prove that

- ☐ *(a)* There exists an integer n such that $P(n, m)$ is false for all integers m.
- ☐ *(b)* There exists integers n,m such that $P(n, m)$ is false.
- ☐ *(c)* For every integer n, and every integer m, the property $P(n, m)$ is false.
- ☐ *(d)* For every integer n, there exists an integer m such that $P(n, m)$ is false.
- ☐ *(e)* For every integer m, there exists an integer n such that $P(n, m)$ is false.
- ☐ *(f)* There exists an integer m such that $P(n, m)$ is false for all integers n.
- ☐ *(g)* If $P(n, m)$ is true, then n and m are not integers.

(10) Let X and Y be statements. Which of the following strategies is NOT a valid way to show that "$X \implies Y$"?

- ☐ *(a)* Assume that X is true, and then use this to show that Y is true.
- ☐ *(b)* Assume that Y is false, and then use this to show that X is false.
- ☐ *(c)* Show that either X is false, or Y is true, or both.
- ☐ *(d)* Assume that X is true, and Y is false, and deduce a contradiction.
- ☐ *(e)* Assume that X is false, and Y is true, and deduce a contradiction.
- ☐ *(f)* Show that X implies some intermediate statement Z, and then show that $Z \implies Y$.
- ☐ *(g)* Show that some intermediate statement $Z \implies Y$, and then show that $X \implies Z$.

(11) Suppose one wishes to prove that "if all X are Y, then all Z are W". To do this, it would suffice to show that

- ☐ *(a)* All Z are X, and all Y are W.
- ☐ *(b)* All X are Z, and all Y are W.
- ☐ *(c)* All Z are X, and all W are Y.
- ☐ *(d)* All X are Z, and all W are Y.
- ☐ *(e)* All Y are X, and all Z are W.
- ☐ *(f)* All Z are Y, and all X are W.
- ☐ *(g)* All Y are Z, and all W are X.

(12) Suppose one wishes to prove that "if some X are Y, then some Z are W". To do this, it would suffice to show that

- ☐ *(a)* All X are Z, and all Y are W.
- ☐ *(b)* Some X are Z, and all Y are W.
- ☐ *(c)* All Z are X, and all Y are W.
- ☐ *(d)* All X are Z, and some Y are W.
- ☐ *(e)* Some Z are X, and some Y are W.
- ☐ *(f)* Some Z are X, and all Y are W.
- ☐ *(g)* All Z are X, and all W are Y.

(13) Let X, Y, Z be statements. Suppose we know that X implies Y, and that Y implies Z. If we also know that Y is false, we can conclude that

- ☐ *(a)* X is false.
- ☐ *(b)* Z is false.
- ☐ *(c)* X implies Z.
- ☐ *(d)* Z is false and X implies Z. Correct Answer. X is false and X implies Z.
- ☐ *(e)* X is false and Z is false and X implies Z.
- ☐ *(f)* None of the above conclusions can be drawn.

(14) [Var. 1] Let X, Y, Z be statements. Suppose we know that X implies Y, and that Z implies X. If we also know that Y is false, we can conclude that

- ☐ *(a)* X is false.
- ☐ *(b)* Z is false.
- ☐ *(c)* Z implies Y.
- ☐ *(d)* Z is false and Z implies Y.
- ☐ *(e)* X is false and Z implies Y.
- ☐ *(f)* X is false, Z is false, and Z implies Y.
- ☐ *(g)* None of the above conclusions can be drawn.

[Var. 2] Let X, Y,Z be statements. Suppose we know that ”X is true implies Y is true”, and ”X is false implies Z is true”. If we know that Z is false, then we can conclude that

- ☐ *(a)* X is false.
- ☐ *(b)* X is true.
- ☐ *(c)* Y is true.
- ☐ *(d)* X is true and Y is true.
- ☐ *(e)* X is false and Y is true.
- ☐ *(f)* X is false, X is true, and Y is true.
- ☐ *(g)* None of the above conclusions can be drawn.

[Var. 3] Let X, Y, Z be statements. Suppose we know that X implies Y, and that Y implies Z. If we also know that X is false, we can conclude that

- ☐ *(a)* Y is false.
- ☐ *(b)* Z is false.
- ☐ *(c)* Z implies X.
- ☐ *(d)* Y is false and Z is false.
- ☐ *(e)* Y is false and Z implies X.
- ☐ *(f)* Y is false, Z is false and Z implies X.
- ☐ *(g)* No conclusion can be drawn.